# Noisy atomic magnetometry in the linear-Gaussian regime

Júlia Amorós Binefa[1], Jan Kołodyński [1]

[1] Centre for Quantum Optical Technologies, Centre of New Technologies, University of Warsaw, Banacha 2c, 02-097 Warsaw, Poland

Continuously monitored atomic spin-ensembles allow, in principle, for real-time sensing of external magnetic fields beyond classical limits. Within the linear-Gaussian regime, thanks to the phenomenon of measurement-induced spin-squeezing, they attain a quantum-enhanced scaling of sensitivity both as a function of time, $t$, and the number of atoms involved, $N$ [1]. In our work, we rigorously study how such conclusions change when inevitable imperfections are taken into account: in the form of collective noise, as well as stochastic fluctuations of the field in time. We prove that even an infinitesimal amount of noise disallows the error to be arbitrarily diminished by simply increasing $N$, and forces it to eventually follow a classical-like behaviour in $t$. However, we also demonstrate that, "thanks" to the presence of noise, in some regimes the model based on a homodyne-like continuous measurement actually achieves the ultimate sensitivity allowed by the decoherence, yielding then the optimal quantum-enhancement. We are able to do so by constructing a noise-induced lower bound on the error that stems from a general method of classically simulating a noisy quantum evolution, during which the stochastic parameter to be estimated—here, the magnetic field—is encoded. The method naturally extends to schemes beyond the linear-Gaussian regime, in particular, also to ones involving feedback and active control.

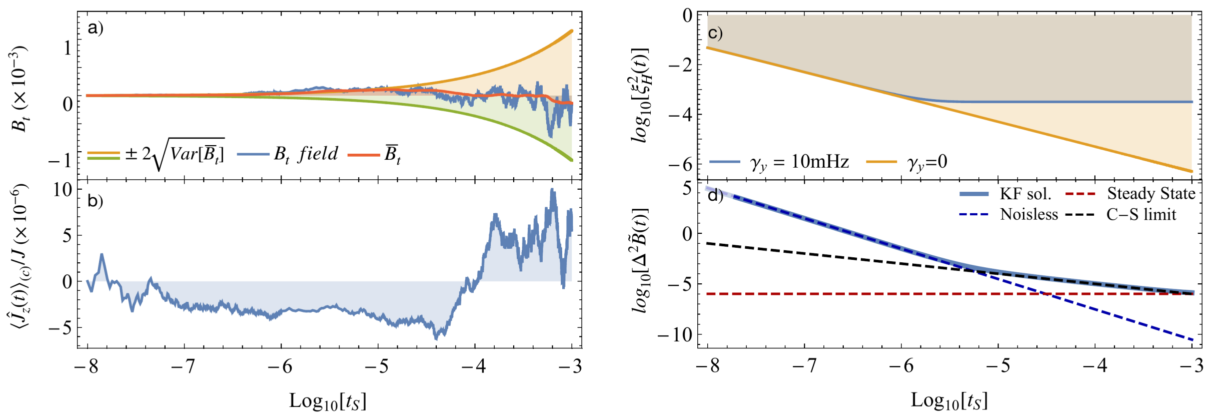


**Figure 1.** (a) An exemplary *stochastic magnetic-field signal* with its fluctuations lying within the confidence interval. (b) Resulting *conditional evolution of the ensemble spin-component* being continuously measured. (c) The *squeezing parameter* being then attained with and without collective decoherence (blue and orange, respectively). (d) The *estimation error* (solid line) of the field at time $t$ converging to the steady-state solution, compared against its noiseless behaviour ($\propto 1/(t^3N^2)$) and the ultimate Classical-Simulation limit ($\propto 1/t$) imposed by the collective decoherence.

[1] J. Geremia, J. K. Stockton, A. C. Doherty and H. Mabuchi, PRL, **91**(25), 250801 (2003).